WEIGHT BIAS IN VALUE WEIGHTED INDEXES OF PHYSICAL PRODUCTION

BY

WILLIAM LLOYD LEAVITT

B.S.C., Temple University, 1936

A.M., Temple University, 1939

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ECONOMICS IN THE GRADUATE SCHOOL OF THE UNIVERSITY OF ILLINOIS, 1942

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The Problem of Weighting and Weight Bias

In constructing an index of the physical volume of production, the problem of weighting the relatives is one of giving each of the commodities represented in the index an influence on the index number which is consistent with the importance of the commodity in the universe which is being measured. The most satisfactory measure of economic importance yet discovered is value. Yet when the relatives of two periods are being averaged for purposes of comparison, since the composition of total value is not the same within each of these periods, the use of different types of value weighting results in index numbers which tend to differ. To the extent to which the value weighting employed does not conform to the composition of total value within each of the periods, the index is in error. If it can be demonstrated that the use of a particular type of value weight results in an index which is consistently lower or higher than it should be, the index may be held as containing weight bias. Bias thus is a forseeable tendency to err in one direction.

Four different varieties of value weights may be employed. These are: I, value in the base period; II, price in the given period times quantity produced in the base period; III, price in the base period times quantity produced in the given period, and; IV, value in the given period. It has been the purpose of this study to ascertain the bias involved in the use of these different systems of value weighting in averaging relatives of physical production.

The problem may be approached in three different ways. First, the formulas themselves may be examined in order to find inherent characteristics of the formulas which cause bias. Secondly, assumptions may be laid down as to the relations between prices and quantities of commodities included in the index. This second approach is a refinement of the first method, since the

first method takes no account of price-quantity correlation. The third approach consists in noting the changing composition of total value as the general business cycle passes through its several phases.

Weight Bias: Examination of Weighting Formulas

Indexes of physical production weighted by base year values tend to be lower than indexes weighted by the product of base year prices and given year physical production. That is, weighting I tends to yield indexes which are lower than those yielded by weighting III, where the same data are used. Further, indexes of physical production, weighted by the product of given year prices and quantities produced in the base period, tend to be lower than indexes weighted by given year values. That is, weighting II tends to yield indexes which are lower than those yielded by weighting IV, where the same data are used.

The explanation for this bias is found in the inverse correlation between the relatives and the weights under weightings I and II, and direct correlation between the relatives and the weights under weightings III and IV. Under weightings I and II the high relatives for the given period are for commodities which were low in amount in the base period. When base period value weights are employed these low quantities, multiplied by price, allow slighter influence to the high relatives. Conversely, under weightings I and II, the low relatives are those of commodities which were large in quantity in the base period. When these large quantities are multiplied by the price component of the weight, the low relatives are accorded a larger influence on the resulting index.

Weightings III and IV, on the other hand, weight the low relatives less and the high relatives more than do weightings I and II. Since weighting systems III and IV each contain given period quantity elements the high production relatives are weighted with values containing large given year quantities. As the low relatives are heavily weighted under I and II, and less heavily weighted under III and IV, and as the high relatives are lightly weighted under I and II, and more heavily weighted under III and IV, indexes weighted by weighting systems I and

II tend to yield a downward bias in the physical production indexes computed thereby, while weighting systems III and IV tend to yield an upward bias, where the same data are used.[1]

Weight Bias: Price-Quantity Correlation

The second approach takes account of the relations between the prices and quantities which comprise the value weights. Here it is found that where price and quantity are directly correlated, that is where high prices are associated with high production relatives and low prices with low production relatives, indexes of physical production computed with weighting II tend to be higher than those obtained with weighting I. Further indexes computed with weighting III tend to be lower than those derived by weighting IV. Here, while weightings I and II continue to yield indexes which have a downward weight bias, and weighting systems III and IV yield indexes which are characterized by an upward weight bias, the range of the variation is marked by the indexes weighted with base and given year values.

If the price-quantity correlation is negative, however, indexes of physical production weighted with base year values tend to be higher than those yielded when weighting II is employed. Further, the use of given year value weights results in indexes which are lower than the index computed with weighting III. Thus, under inverse price-quantity correlation, the indexes of physical production constructed with weighting systems II and III mark the range of the averages yielded. As before, however, weighting systems I and II are characterized by a downward bias, and III and IV by an upward weight bias.

By the analysis of price and production variation through the general business cycle, it is found that high prices tend to be associated with small quantities and low prices with large quantities. When commodities are classified with respect to price and production flexibility to the cyclical forces it is found that

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[1] As has been noted in this study, the contrasting weight bias of systems I and III, and between II and IV, for physical production index numbers, is the same as the weight bias characterizing price indexes as developed by Irving Fisher in *The Making of Index Numbers*, Houghton Mifflin Co., Cambridge, Massachusetts, 1922.

commodities classified as durable, producers' and manufactured tend to vary more in production and less in price than commodities classified as nondurable, consumers' and raw. Of these three factors the classification on the basis of durability is found to be more clearly indicative of price and production responses to cyclical factors than are classifications on the basis of type of user and state of fabrication.

The cyclical pattern of price and production, both as indicated by economic analysis and as evinced by reference to the behavior of price and production relatives over the 1929-1937 cycle, is one of inverse price-quantity correlation. Accordingly it is to be expected that indexes of physical production computed for the period 1929-1937 would contain the weight bias indicated under inverse price-quantity correlation. The sample indexes of physical production computed for this period follow this pattern. That is, indexes yielded when weighting I is employed, are greater than those obtained by using weighting system II. Likewise III is greater than IV and IV is greater than I.

Weight Bias: Cyclical Changes in the Composition of Total Value

The last approach takes account of the changing composition of total value through the several phases of the general business cycle. It is noted that durable goods and producers' goods are a larger proportion of total value during the prosperity phase of the trade cycle than at any other position of the cycle. Further, during the depression phase of the cycle, the value of commodities belonging to these classifications tends to be a smaller proportion of total value than is the case in any other stage of the cycle. On the other hand, consumers' and nondurable goods are a smaller proportion of total value in prosperity and a larger proportion in depression than during other cyclical stages.

These cyclical changes in the composition of total value are of direct influence on indexes of physical production where value weights are used. When base period value weights are employed, and when the base period is one in which cyclical disturbance is mild, the index computed will tend to be lower than it should be—it will have a downward weight bias. In the trough of the cycle the production relatives which have declined most are those

for the producers' and durable groups. Since these are also the groups which have declined most as a proportion of total value, base period value weighting overweights these low relatives. Where given year value weighting is employed these low relatives receive slighter value weights and the decline is understated. Accordingly, physical production indexes computed with weighting I tend to be less than those obtained by using weighting IV, when the given period is one of depression.

If the given period is one of prosperity, the durable and producers' commodities tend to have the higher production relatives. As these groups are a greater proportion of total value during this phase of the cycle than in other stages of the cycle of general business, given year value weighting tends to overstate the rise by allowing heavier weights to those relatives which have risen most. Conversely, base period value weighting understates the rise by weighting the high relatives less heavily than weighting IV and the low relatives more heavily than weighting IV. Thus, owing to the nature of cyclical changes in the composition of total value, weighting I tends to yield indexes which are lower, and weighting IV indexes which are higher than they should be. Physical production indexes which employ weighting I are characterized by a downward weight bias, while those using weighting IV are characterized by an upward weight bias.

Isolation and Removal of Weight Bias

Since an arithmetic average of relatives contains an upward type bias, and, depending upon the weights employed, either upward or downward weight bias, the bias due to the weights may not be measured until type bias has been removed. As a geometric average of relatives is free from type bias, relatives averaged geometrically contain only weight bias. The weight bias present in arithmetic averages of relatives weighted by the four systems of value weighting is also present in geometric averages with like weights. Since averages of relatives weighted by base year values are characterized by a downward weight bias, and averages weighted by given year values are characterized by an upward weight bias, the square root of the product of geo-

metric averages of relatives with weightings I and IV will yield an index with neither type nor weight bias. Likewise geometric averages of relatives with weighting systems II and III may be crossed geometrically to eliminate weight bias. This follows as these weighting patterns are characterized by opposite weight biases while the average employed does not introduce type bias. Indexes crossed in this manner yield substantially the same results as those yielded by Fisher's "ideal" index, where the same data are used. The "ideal" index is somewhat simpler to compute than the cross weighted geometric average of relatives, but the process of removing bias from the latter form of index is more convincing. The geometric average, however, has the disadvantage that if one of the series being averaged has a value of zero for the given year, the index is indeterminate.

Where precision is required of the formula used in averaging physical production relatives, a geometric crossing of the geometric average of relatives weighted by base and by given year values, is recommended. This formula may be written

$$Q = \left[\left(\Pi \left(\frac{qn}{qo}\right) Vo\right)^{\frac{1}{\Sigma Vo}} \cdot \left(\Pi \left(\frac{qn}{qo}\right) Vn\right)^{\frac{1}{\Sigma Vn}} \right]^{\frac{1}{2}}$$

Where, Q is the quantity index.

ΣVo is the sum of the value of the commodities in the base period.

ΣVn is the sum of the value of the commodities in the given period.

Π means the product of such terms as.

Vo is the value of the quantity produced in the base period.

Vn is the value of the quantity produced in the given period.

qo and qn refer to the quantity produced in the base and given periods respectively.

For general purposes, however, such a complicated procedure need not be followed. The weighted arithmetic average of relatives, weighted by base period values, is usually a satisfactory index. In this method of averaging, the upward type bias of an arithmetic average of relatives tends to compensate for the downward bias of the weights. The nearer the value relationships of the base period are to being an average for the entire period covered by the index, the less will be the weight bias. With base period weighting, weight bias is appreciable only if the composition of total value differs widely between the periods under comparison. Recognition of this downward bias and the circumstances under which the error is increased, emphasize the necessity of periodic revision of the base period.

Weighting by base period values, or by fixed weights, has the further advantage of inter-comparability among the indexes computed. If given year value weighting is used, or if the weighting includes given year prices or quantities, each year is strictly comparable only with the base year. Where fixed base weighting is used, since all the years employ the same weights, the values of the index for the various years may be compared.

The problem of bias in index numbers is only one of several pitfalls in obtaining a number representative of composite variation. The behavior of the series chosen for the index must be representative of the pattern followed by the universe from which these data have been selected. Care must be used to maintain comparability and mistakes must be avoided in computation. When this is done, the reduction or elimination of bias in the formula used attains significance.

VITA

William Lloyd Leavitt was born in Nashua, New Hampshire, September 28, 1914. He attended Temple University from 1932 until 1936, majoring in Economics. In 1936 he was graduated by Temple University with the degree of Bachelor of Science in Commerce. He was employed by the Girard Trust Company of Philadelphia, in the Trust Accounting Department from 1936 until 1939. From 1937 until 1939 he undertook part time graduate work, majoring in Economic Theory, and acquired the A.M. degree from Temple University in June 1939. He wrote a Master's Thesis entitled "An Economic Analysis of the Old-Age Reserve Fund of the Social Security Act of 1935." From September 1939 until May 1942 he held an assistantship in the Department of Economics of the University of Illinois, teaching Economics and Statistics and engaging in graduate work. A minor was taken in Statistics and a major in Economic Theory. He is a member of Pi Gamma Mu and Artus, (Omicron Delta Gamma).